

UDC 624.154 : 624.131

The improved soil base model for the calculation of the combined raft pile foundation with the structural nonlinear behavior of the elements

Samorodov Oleksandr^{1*}, Tabachnikov Serhii², Dytiuk Oleksii³, Bondar Oleksandr⁴

¹O.M. Beketov National University of Urban Economy in Kharkiv; <https://orcid.org/0000-0003-4395-9417>

²O.M. Beketov National University of Urban Economy in Kharkiv; <https://orcid.org/0000-0002-2619-8612>

³O.M. Beketov National University of Urban Economy in Kharkiv; <https://orcid.org/0000-0001-8363-6132>

⁴O.M. Beketov National University of Urban Economy in Kharkiv; <https://orcid.org/0009-0006-8468-6805>

*Corresponding author E-mail: osamorodov@ukr.net

The paper considers and theoretically justifies the improved model of the soil base for the combined raft pile foundation (CRPF) to consider the nonlinear behavior of the elements “after” and “before” the connection between the raft and the piles (structural nonlinearity) to calculate the stress-strain state using the finite element method in present-day calculation packages. Using the improved model makes it possible to qualitatively simulate the behavior of the CRPF with the structural nonlinearity in the behavior of its elements. The simulation and numerical calculation of the base-CRPF system was performed using a specific example and it was found that the application of the proposed model with consideration to the structural nonlinearity of the behavior of the foundation elements reduces the moment forces in the raft to 15% in comparison with the application of the full load at once and the behavior of the raft with the permanent connection between the raft and the piles

Keywords: soil base, model, combined raft pile foundation, methodology, stress-strain state

Удосконалена модель ґрунтової основи для розрахунку комбінованого плитно-пального фундаменту з конструктивної нелінійності роботи елементів

Самородов О.В.^{1*}, Табачников С.В.², Дитюк О.Є.³, Бондар О.П.⁴

^{1, 2, 3, 4} Харківський національний університет міського господарства ім. О.М. Бекетова

*Адреса для листування E-mail: osamorodov@ukr.net

У статті пропонується та теоретично обґрунтовується удосконалена модель ґрунтової основи комбінованого плитно-пального фундаменту для врахування нелінійної роботи його елементів «до» та «після» з'єднання плити та паль (конструктивна нелінійність) для розрахунку методом скінченних елементів напружено-деформованого стану у сучасних розрахункових комплексах. Використання удосконаленої моделі дає змогу якісно моделювати процес поведінки КППФ з конструктивною нелінійністю роботи його елементів. Результатом є отримання надійних результатів щодо напружено-деформованого стану системи «основа – фундамент – споруда». На конкретному прикладі виконано моделювання та чисельний розрахунок системи «основа – КППФ» із використанням лінійно-пружної моделі ґрунту та нелінійної моделі Мора-Кулона. Аналіз отриманих даних показує, що різниця у результатах складає не більше 2%. За критерій оцінки впливу запропонованої комбінованої моделі ґрунтової основи при розрахунках різних фундаментів прийнято суму згинальних моментів уздовж плити $\Sigma|M_x|$. Встановлено, що врахування 2-х етапного формування НДС КППФ із застосуванням запропонованої моделі зменшує моментні зусилля у плиті на 2-му (останньому) етапі до 15% у порівнянні з прикладанням одразу повного навантаження і роботою плити у якості ростверку з постійним з'єднанням плити та паль (один етап). За результатами розрахунків встановлено, що при прийнятті плитною частиною 100% навантаження та інших рівних умовах моментні зусилля у плиті завжди менші ніж у випадку з'єднання паль з плитою, що відбувається через відсутність значної концентрації зусиль у кутових та периферійних палях у разі роботи плити як ростверку

Ключові слова: ґрунтова основа, модель, комбінований плитно-пальовий фундамент, методика, напружено-деформований стан

Introduction

In modern geotechnics, with the advances of information technology and the availability of powerful software packages for the calculation of the entire base-foundation-building (BFB) system, one of the main research areas is the development, improvement and research of soil base models to ensure adequate interaction between the components of the system during the construction and operation of structures (buildings).

It is generally known that to obtain reliable and valid results of the stress-strain state calculation for the foundation designs in the BFB system, a model of the soil base with appropriate parameters should be chosen that is close to the behavior of the actual soil medium by two criteria: distribution capability and deformability of the foundations of buildings.

Review of the research sources and publications

Back in the last century, the model of the soil base in the form of a continuous linear elastic layer was widely used in engineering calculations of bases and foundations, as it was provided by the national building code [1, 2] and required only setting the thickness H of the layer (compressible thickness) and the stress-strain properties of the soil (the total strain modulus E and

Poisson's ratio ν). Furthermore, this analytical model had no constraints in plan. Today, with the expansive growth of information technology, when simulating and performing numerical calculations of the BFB system in the powerful calculation packages such as SOFiSTiK, ABAQUS, Plaxis, SCAD, Lira and others, the three-dimensional formulation generally uses a soil base model in the form of a *continuous layer of finite distribution capability* (Fig. 1) [12-22] (the concept is introduced for the first time), which, in addition to the vertical strain constraints at a certain depth H , also has the horizontal strain constraints at a certain distance from the load in plan ($L_x \times L_y$).

These boundary conditions for the model are based on the fact that under the action of external loads on the soil base a spatial stress-strain region is formed, beyond which the soil strains can be neglected, since the additional load at the boundaries of the soil mass does not exceed the structural strength of soil [3]. In addition, any patterns of soil straining under loads, including time patterns, can be specified for the model itself. For two-dimensional formulations (plane strain) the model is well-known as the model of a continuous layer of finite width.

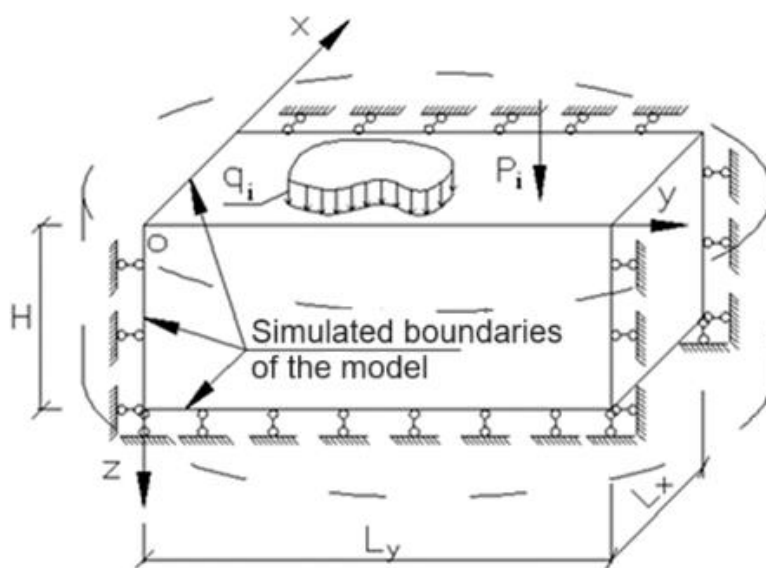


Figure 1 – Soil base model in the form of a continuous layer of finite distribution capability (for three-dimensional problems)

Definition of unsolved aspects of the problem

The major drawback of the existing approaches to simulating the interaction between the building and the soil base using classical models is that it is impossible to incorporate the structural nonlinearity of the behavior of the BFB system, particularly for a new type of high-performance large-size combined raft pile foundations for multistory and high-rise buildings [4, 22], where no contact between the piles and the raft is observed at the first stage of loading. A utility model of a combined raft pile foundation (CRPF) [1] (Fig.2) has been patented, which consists of the raft 1 and the piles 2 with the diameter d , with the gap 3 with the height Δ

being provided between the raft and the piles. For the technological convenience of ensuring that no contact is made between the raft and the pile heads within the concrete bed 4, the gap under the raft can be filled up with a low-modulus material. The soil base is designated as 5.

Problem statement

The purpose of this work is to improve and theoretically justify the soil base model and the methodology for identifying its parameters to calculate a CRPF in the BFB system.

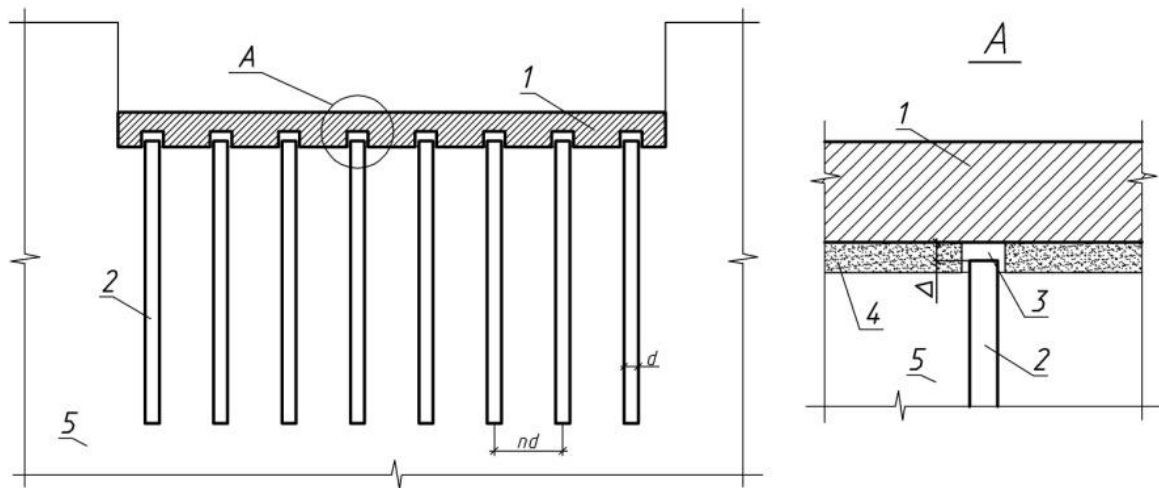


Figure 2 – General arrangement of a combined raft pile foundation (CRPF)

Basic material and results

To represent a soil base as a linear strain medium under conditions of a single load application, which does not result in significant growth of the regions of the ultimate stress state (“unstable” regions), it is required to ensure the normative condition [1, 2]:

$$p \leq R, \quad (1),$$

where p is the average pressure under the bottom of the foundation, kPa;

R is the design resistance of the soil of the foundation base, kPa.

Since this paper considers large-size combined raft pile foundations, the average pressure under the bottom of the raft of the foundation does not generally exceed the design resistance of the soil of the base, that is to say it is appropriate to adopt the linear strain law for the soil base under load, which was proposed by N. Gersevanov [6] and V. Florin [7].

Following the previous solutions to test problems on various interactions between the raft and the piles in present-day calculation packages (SOFiSTiK, PLAXIS), we propose an improved model of the soil base for the CRPF and a methodology for simulating the base-CRPF subsystem to incorporate the structural nonlinearity of the behavior of the CRPF, that is, the behavior of the foundation elements “before” and “after” the connection of the piles to the raft.

An improved model of the soil base in the form of a combination of a continuous linear strain layer of finite distribution capability and a Winkler-Fuss layer is shown in Fig. 3.

The methodology for simulating the base-CRPF subsystem includes the stages as follows:

1. We simulate the soil base of the CRPF with the physical and mechanical properties of soil layers and model dimensions such as compressible thickness H_{ppl} and overall dimensions $L_x \times L_y$ in plan, and corresponding vertical and horizontal strain constraints on the boundaries of the model.

2. We simulate the interaction between the soil base and a single pile separately (or using a built model). Based on the calculations and results of soil tests with piles at the construction site, we iteratively determine the connection stiffness G_p under the lower end of a single pile. In this case, the connection stiffness G_p of a single pile can be either linear or nonlinear (for example, bilinear). To determine the stiffness of piles in the pile field, we should consider their interaction; therefore, the stiffness G_{pf} under the lower ends of the piles will be equal to:

$$G_{pf} = G_p \cdot \zeta [\text{kN/m}],$$

where G_p is the stiffness of the connection under the lower end (bottom) of a single pile, kN/m;

ζ is the coefficient of transition from the settlement of a single pile to the settlement of the pile field, units. We take the normative value of $k = 0.2$ or when justified, the value of $k = 0.25 \div 0.33$ can be taken [8].

3. We simulate a CRPF with no contact between the raft and the piles with the gap Δ between them.

4. We simulate special inserts with the thickness Δ between the raft of the foundation and the piles, the stiffness of which should be not less than that of the foundation elements (by convention, “concrete” inserts). The “concrete” inserts should provide a connection between the raft of the foundation and the piles at the stage of calculation “after” the connection to the raft.

Next, we simulate the superstructure with the appropriate effective, wind and other loads on it to obtain a model of the entire base-CRPF-building system (Fig. 3).

To calculate the structural nonlinearity of the combined raft pile foundation, the main calculation steps are as follows:

- We determine the stress-strain state of the **raft** of the foundation “before” the connection to the piles. It is determined for the part of the vertical load p_{pl} , which is taken by the raft of the foundation “before” the connection to the piles;

– We determine the stress-strain state of the **raft** of the foundation “**after**” the connection to the piles to find the most unfavorable combination of loads on it. It is determined for the additional (effective) vertical load p_{ad} after developing the stress-strain state at the previous stage.

To investigate the effect of the proposed model of the soil base on the stress-strain state of the CRPF, it is proposed to consider a simple example of calculation with the initial data of an actual construction project in a two-dimensional formulation:

– The overall normative vertical average load under the raft of the combined raft pile foundation is $p_{tot}=p_{pl}+p_{ad}=167$ kPa;

– The gap between the raft and the piles is $\Delta=0.05$ m=5.0 cm;

– The soil base takes the average vertical load under the raft of $p_{pl}=119$ kPa (approximately of the weight of

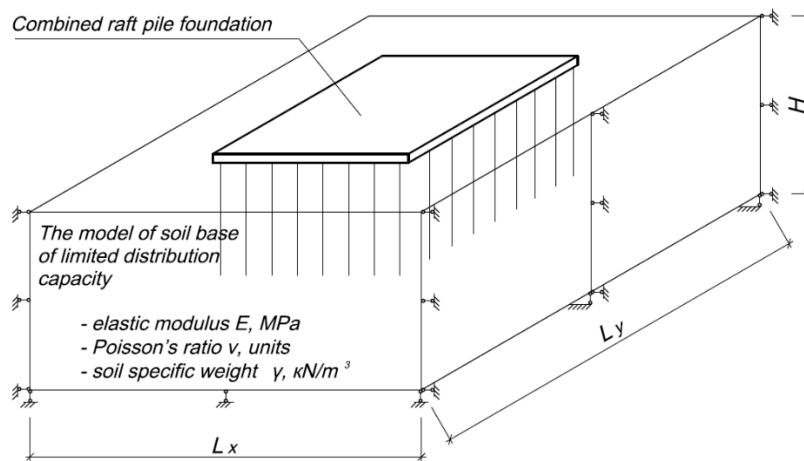
the total building volume) “before” the connection to the piles (Stage 1);

– The soil base takes the additional average vertical load under the raft of $p_{ad}=48$ kPa (effective load) “after” the connection to the piles (Stage 2);

– The linear stiffness under the ends of the piles is $G_{pf}=32000$ kN/m, which is determined from numerical iterative calculations of the interaction between the soil mass and the single pile under the action of a vertical force $F=1200$ kN on the single pile with the stiffness under the lower end of $G_p=160000$ kN/m (determined iteratively) and its settlement ≈ 5 mm, which corresponds to the results of field tests of soils with bored piles [10].

The conditional calculation patterns of the base-CRPF system are shown in Figs 4 and 5.

a)



b)

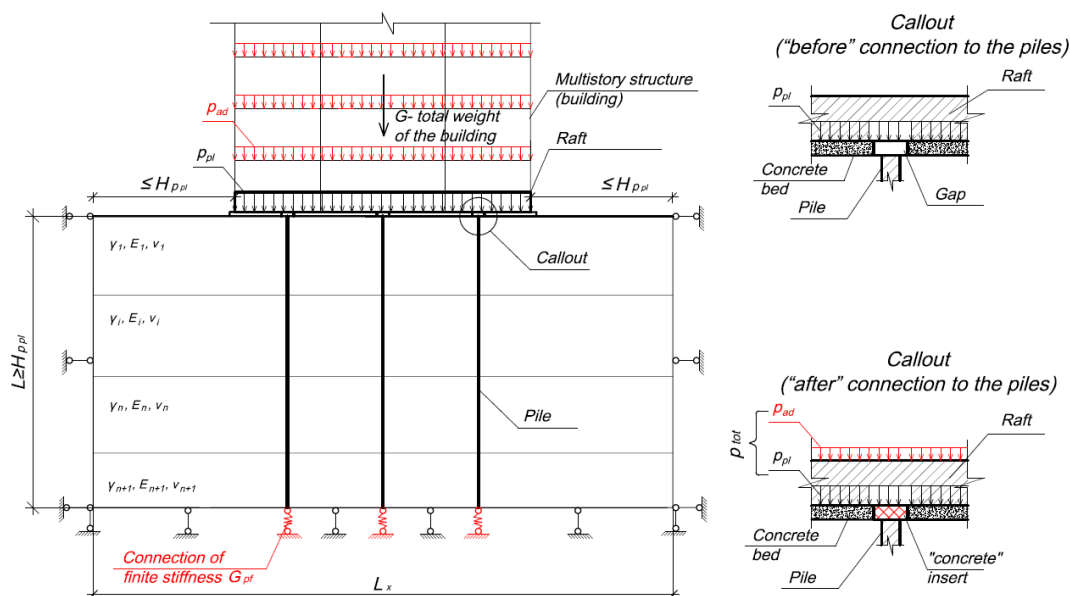


Figure 3 – General arrangement of an improved model of the soil base for the CRPF:

a) Three-dimensional view of the base-CRPF subsystem; b) View of the base-CRPF-building system with the specific locations of the connections of finite stiffness G_{pf} under the bottom of the piles.

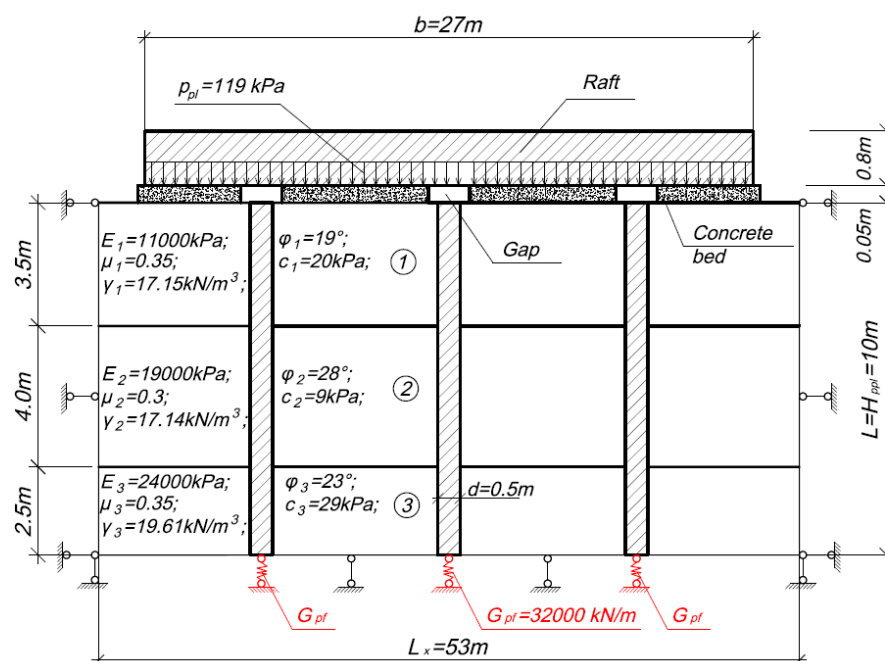


Figure 4 – Conditional calculation pattern of the base-CRPF system “before” the connection to the piles (Stage 1).

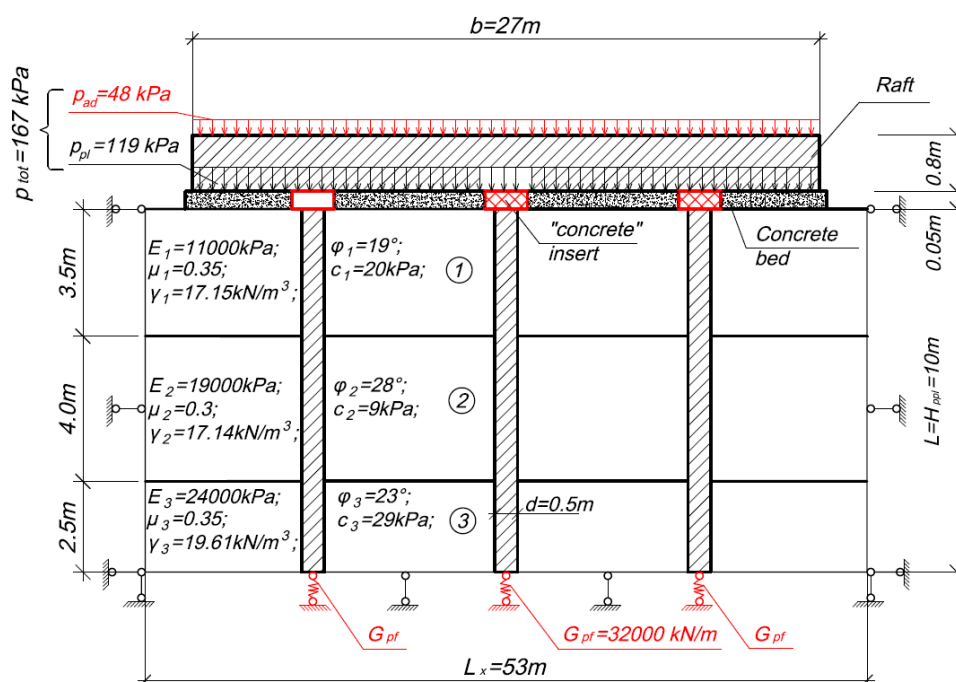


Figure 5 – Conditional calculation pattern of the base-CRPF system “after” the connection to the piles (Stage 2).

To investigate the performance of the proposed model of the soil base, a finite element model of the base-CRPF subsystem, which consists of a soil base and a combined raft pile foundation, was created using the PLAXIS 3D package according to the calculation patterns of Figs 4 and 5 (Fig. 6) [11].

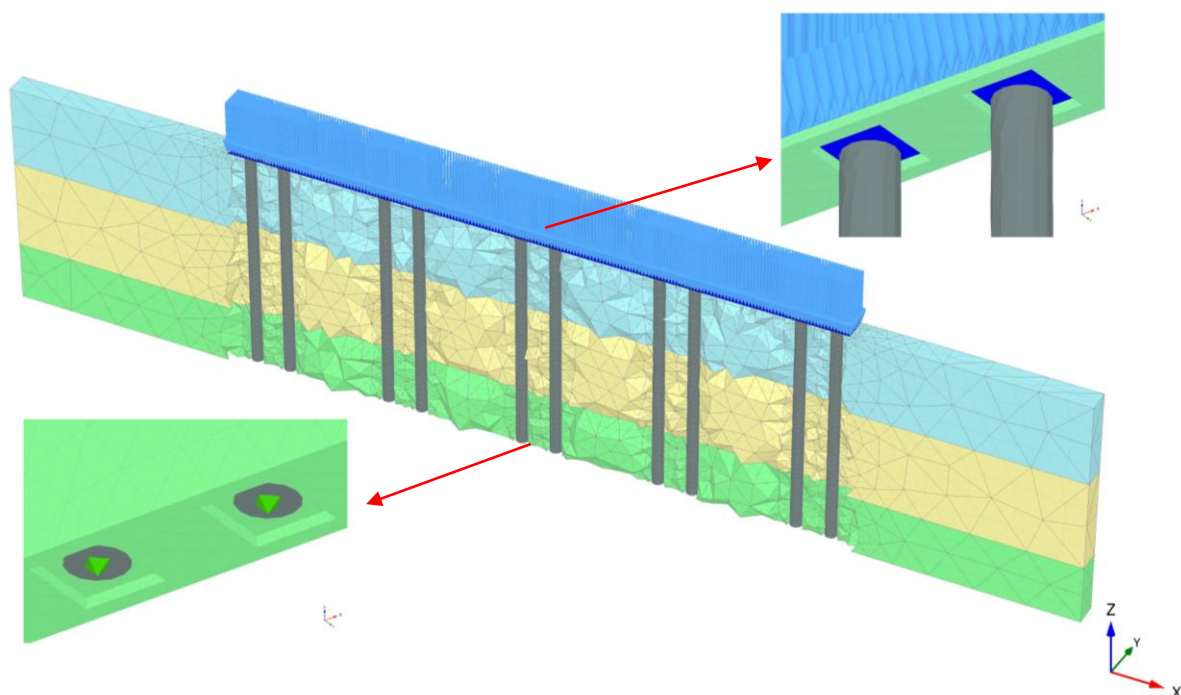


Figure 6 – General view of the base-CRPF system in a two-dimensional formulation.

The soil base and piles were simulated using a linear-elastic material model which is based on Hooke's law for isotropic elasticity and represented by solid finite elements with consistent stress-strain properties such as Young's modulus (elastic modulus) E in kN/m^2 and Poisson's ratio ν in units.

The raft is simulated by plate finite elements using an elastic material model with the parameters as follows: specific weight γ in kN/m^3 , elastic modulus E in kN/m^2 , and Poisson's ratio ν in units.

Fixed-end anchor elastic elements with the elastic modulus E (kN/m^2), cross-section A (m^2), and thickness Δ (m), that is, with the linear stiffness of $G_{pf}=E \cdot A / \Delta = 32000 \text{ kN/m}$, are used as the connection of finite stiffness under the lower ends of the piles.

A two-row arrangement of the piles with the spacing $3d$ at a distance of 6.0 m between the rows was adopted.

The load is assumed to be evenly distributed over 1 (one) running meter of the raft.

Fig. 7 below shows the results of the preliminary simulation of the interaction between the soil base and the single pile to determine iteratively the connection stiffness G_p under the bottom end of the single pile. Iterative calculations were performed for both the linear-elastic model and the nonlinear Mohr-Coulomb model for soil. The difference was not more than 2%.

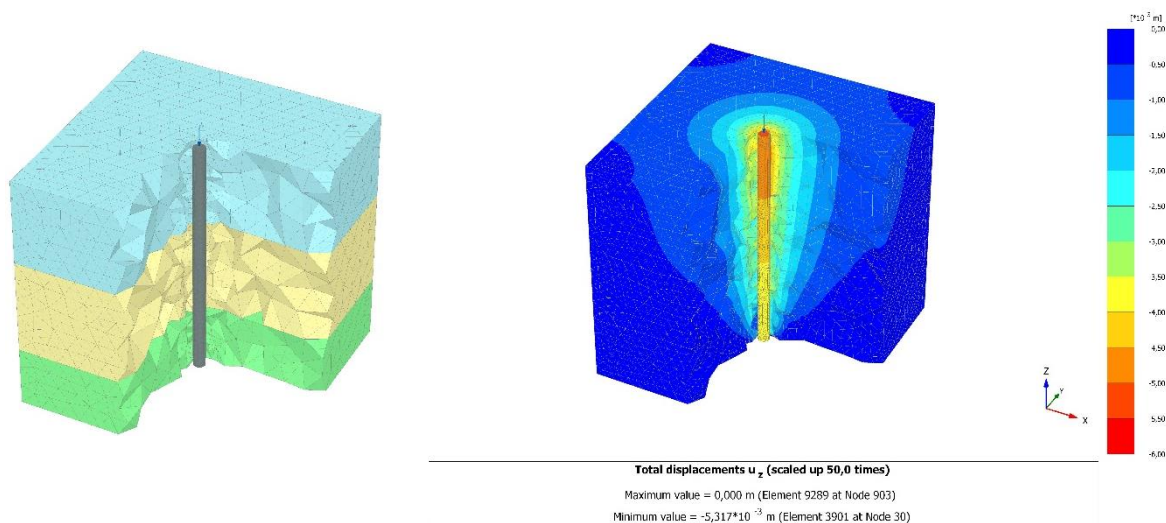


Figure 7 – General view and straining of the base-single pile system.

Fig. 8 shows the bending moment curves along the raft of different foundations:

- CRPF (2 stages) L-E model: calculation pattern for the base-CRPF system “after” the connection to the piles (Stage 2), where the soil base is simulated using a linear-elastic material model;
- CRPF (2 stages) M-K model: calculation pattern for the base-CRPF system “after” the connection to the piles (Stage 2), where the soil base is simulated using the nonlinear Mohr-Coulomb material model;
- RPF L-E model: calculation pattern for the base-RPF system with the full load p_{tot} being applied and the behavior of the raft as a raft with the permanent connection between the raft and the piles, where the soil base is simulated using a linear-elastic material model.

behavior of the raft as a raft with the permanent connection between the raft and the piles, where the soil base is simulated using a linear-elastic material model;

- RPF M-K model: calculation pattern for the base-RPF system with the full load p_{tot} being applied and the behavior of the raft as a raft with the permanent connection between the raft and the piles, where the soil base is simulated using the nonlinear Mohr-Coulomb material model.

The criterion for evaluating the effect of the proposed combined model of the soil base in the calculation of various foundations is the sum of the bending moments along the raft: $\Sigma|M_x|$, kN·m (Fig. 9).

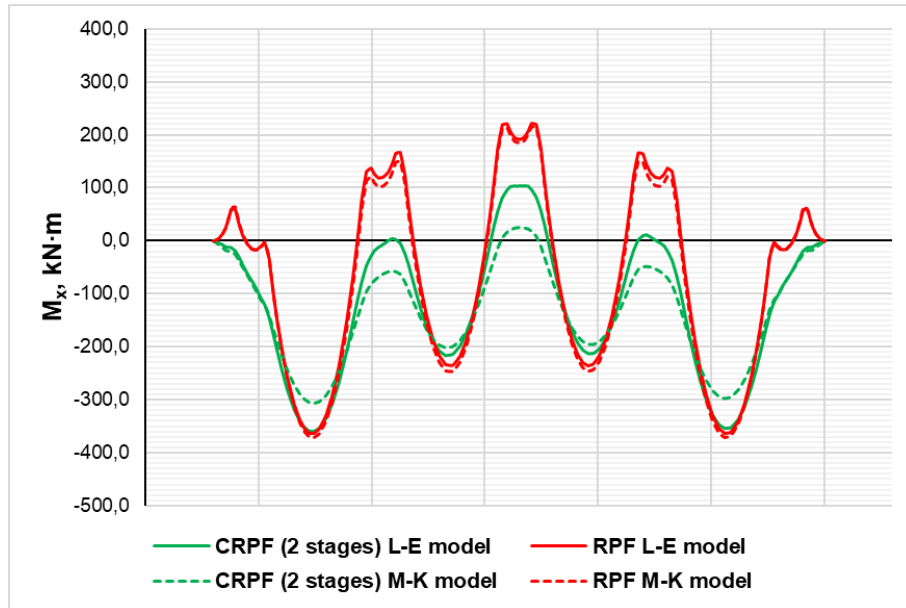


Figure 8 – Bending moment curves M_x along the raft, kN·m

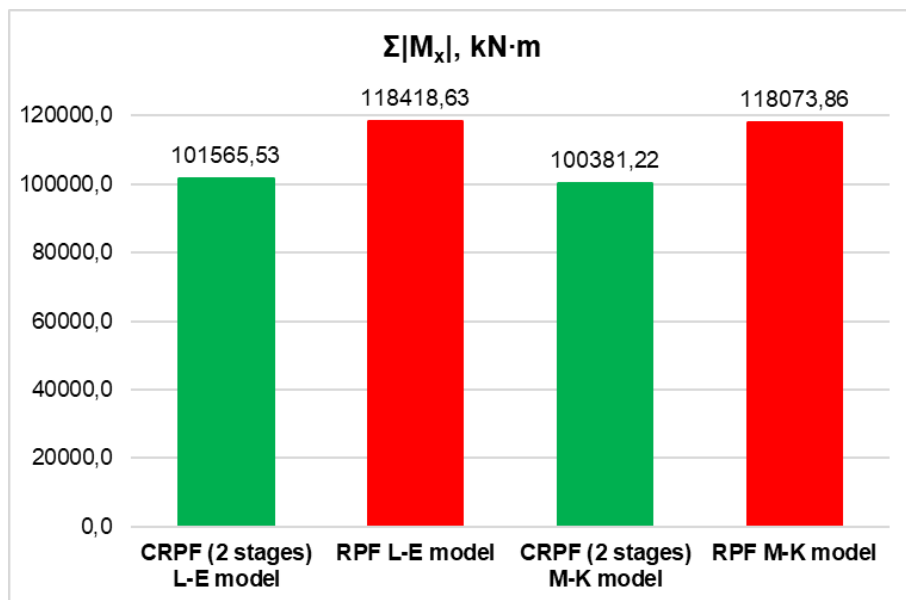


Figure 9 – Sum of the bending moments along the raft: $\Sigma|M_x|$, kN·m

It should be noted that the calculation of a 2-stage development of the stress-strain state of the CRPF reduces the moment forces in the raft at the 2nd (last)

stage to 15% in comparison with the application of the full load $p_{tot}=p_{pl}+p_{ad}=167$ kPa and the behavior of the

raft as a raft with the permanent connection between the raft and the piles.

The analysis of the stress-strain state of the combined raft pile foundation at different stages of the interaction with the proposed soil base model helps to confirm the physical significance of this structural nonlinearity of the behavior of the foundation elements. It should be noted that in this case there is no point in comparing the stress state of the raft when using other soil base models since this is the only available model now that can qualitatively simulate the behavior of the CRPF with the structural nonlinearity in behavior.

Conclusions

The conducted investigations suggest the conclusions as follows.

1. A soil base model in the form of the combination of a continuous linear strain layer of finite distribution capability and a Winkler-Fuss layer was improved and theoretically justified. A methodology for simulating the base-CRPF subsystem was developed to determine the stress-strain state of the CRPF in present-day calculation packages. In this respect, the improved model is capable to qualitatively simulate the behavior of the CRPF with the structural nonlinearity in the behavior

of its elements, raft and piles, as shown in a specific example.

2. Numerical studies of the model's impact on the distribution of bending moments in the raft of various foundations were conducted, which showed a decrease in moment forces in the raft of up to 15% when considering the structural nonlinearity of the foundation elements in comparison with the application of the full load p_{tot} at once and the behavior of the raft as a raft with the permanent connection between the raft and the piles.

3. The results of the calculations of separately performed test tasks show that when the raft takes 100% of the load and other conditions are equal, the moment forces in the raft are always less than in the case of the pile-raft connection, which makes the physical significance owing to the absence of a significant concentration of forces in the corner and peripheral piles in case the raft behaves as a raft.

4. The improved combined model may also be used to calculate classical piled raft foundations to reduce the concentration of forces in the corner and peripheral piles, which requires further research into the proposed soil base model.

References

1. ДБН В.2.1-10:2018 (2018). *Основи і фундаменти будівель та споруд. Основні положення*. Київ: Міністерство регіонального розвитку, будівництва та житлово-комунального господарства України, ДП «Укрархбудінформ»
2. Лучковский И.Я. (2000). *Взаимодействие конструкций с основанием*. Харків: ХДАГХ (Бібліотека журналу ITE)
3. Ter-Martirosyan, Z. G. & Ter-Martirosyan, A. Z. (2009). Soil beds of high-rise buildings. *Soil Mechanics and Foundation Engineering*, Vol. 46, No. 5, 165-179. <http://doi.org/10.1007/s11204-009-9067-7>
4. Самородов А.В. (2017). *Проектування ефективних комбінованих пальових і плитних фундаментів багатоповерхових будівель: монографія*. Харків: «Друкарня Мадрид»
5. Комбінований плитно-пальовий фундамент : пат. 148444 Україна : МПК E02D 27/12 (2006/1) / Самородов О. В., Дитюк О. Є., Муляр Д. Л., Табачников С. В. № u 2020 07173 ; заявл. 09.11.2020 ; опубл. 11.08.2021, Бюл. № 32
6. Герсевич Н.М. (1930). Досвід застосування теорії пружності до визначення допустимих навантажень на ґрунт на основі експериментальних робіт. *Праці МІІТ*, Вип. XV., 4-11
7. Флорін В.А. (1959). *Основи механіки ґрунтів*. Т. 1. Будвидавництво
8. Бойко І.П., В.Л. Підлутський (2015). Дослідження перерозподілу зусиль у фундаменті при різних варіантах розташування паль. *Основи та фундаменти: Міжвідомчий науково-технічний збірник*, Вип. 37, 64-73
9. Самородов О.В. та ін. (2023). Вплив граничних умов на розподільчу здатність та деформативність моделі ґрунтової основи у вигляді лінійно-деформованого шару скінченної ширини. *Наука та будівництво*, №2 (36), 12-19. <https://doi.org/10.33644/2313-6679-2-2023-2>
10. Самородов О.В., Дитюк О.Є. та Табачников С.В. (2022). Натурні дослідження початкових осідань паль, які
1. DBN V.2.1-10:2018 (2018). *Bases and foundations of buildings and structures. Main provisions*. Kyiv: Ministry of Regional Development, Construction, and Housing of Ukraine, State Enterprise Ukrarkhbudinform
2. Luchkovsky I.Ya. (2000). *Interaction of structures with the base*. Kharkiv: Kharkiv State Academy of Urban Economy (Library of ITE Journal).
3. Ter-Martirosyan, Z. G. & Ter-Martirosyan, A. Z. (2009). Soil beds of high-rise buildings. *Soil Mechanics and Foundation Engineering*, Vol. 46, No. 5, 165-179. <http://doi.org/10.1007/s11204-009-9067-7>
4. Samorodov A.V. (2017). *Designing high-performance combined piled and raft foundations of multistory buildings: monograph*. Kharkiv: Madrid Printing House
5. Samorodov O.V., Dytyuk O. Ye., Mulyar D.L., and Tabachnikov S.V., Utility model patent No. 148444, Ukraine IPC E02D 27/12 (2006/1). COMBINED RAFT PILE FOUNDATION. Application dtd November 09, 2020. Publ. August 11, 2021. Bull. No. 32
6. Gersevanov N.M. (1930). Experience in applying the theory of elasticity to determining permissible loads on soil based on experimental work. *Proceedings of MIIT*, Vol. XV, pp. 4-11
7. Florin V.A. (1959). *Fundamentals of soil mechanics*. T. 1. Budvidavnitstvo, 1959
8. Boyko I.P. & Pidlutskyi V.L. (2015). Study of the redistribution of forces in the foundation with different options for the arrangement of piles. *Bases and foundations: Interdepartmental scientific and technical collection*, Vol. 37, pp. 64-73
9. Samorodov O.V. et al. (2023). The influence of boundary conditions on the distribution capability and deformability of the model of the soil base in the form of a linearly deformed layer of finite width. *Nauka ta budivnytstvo*, No. 2 (36), pp. 12-19. <https://doi.org/10.33644/2313-6679-2-2023-2>
10. Samorodov O.V., Dityuk O.Ye. and Tabachnikov S.V., (2022). Field studies of the initial settlement of piles, which are not connected to the raft, as part of a combined raft pile

не з'єднані з плитою, у складі комбінованого плитно-пальового фундаменту. *Український журнал будівництва та архітектури*, №6 (012), 90-98.

<https://doi.org/10.30838/J.BPSACEA.2312.271222.90.915>

11. Самородов О.В. та ін. (2021). Методика моделювання початкових осідань паль у складі комбінованого плитно-пальового фундаменту в програмному комплексі «PLAXIS 3D FOUNDATION». *Науковий вісник будівництва*. Том 105, №3, 106-114

12. Comodromos, E. M., Papadopoulou, M. C. & Laloui, L. (2016). Contribution to the design methodologies of piled raft foundations under combined loadings. *Canadian Geotechnical Journal*, Vol. 53 (4), 559–577.

<http://doi.org/10.1139/cgj-2015-0251>

13. Chow, H. S. (2007). *Analysis of Piled-Raft Foundations with Piles of Different Lengths and Diameters*. Sydney: The University of Sydney.

<http://doi.org/10.1201/9781439833766.ch84>

14. Cunha, R. P., Poulos, H. G., & Small, J. C. (2001). Investigation of Design Alternatives for a Piled Raft Case History. *Journal of Geotechnical and Environmental Engineering*, 635-641.

[http://doi.org/10.1061/\(ASCE\)1090-0241\(2001\)127:8\(635\)](http://doi.org/10.1061/(ASCE)1090-0241(2001)127:8(635))

15. Hain, S., & Lee, I. (1978). The Analysis of Flexible Raft-Pile Systems. *Geotechnique*, 28 (1), 65-83.

<https://doi.org/10.1680/geot.1978.28.1.65>

16. Poulos, H. (1994). An Approximate Numerical Analysis of Pile-Raft Interaction. *Int. J. NAM Geomechs*, 18, 73-92.

<https://doi.org/10.1002/nag.1610180202>

17. Poulos, H. (2001). *Methods of Analysis of Piled Raft Foundations*. International Society of Soil Mechanics and Geotechnical Engineering

18. Reul, O., & Randolph, M. (2003). Piled Rafts in Over-consolidated Clay: Comparison of In situ Measurements and Numerical Analyses. *Geotechnique*, Vol. 53, No. 3, 301-315.

<https://doi.org/10.1680/geot.2003.53.3.301>

19. Shen, W., Chow, Y., & Yong, K. (1999). A Variational Solution for Vertically Loaded Pile Groups in an Elastic Half-space. *Geotechnique*, Vol. 49, No. 2, 199-213.

<https://doi.org/10.1680/geot.1999.49.2.199>

20. Simeneh Abate (2009). *Analysis and Parametric Study of Piled Raft Foundation Using Finite Element Based Software*. Addis Ababa University

21. Yunfei Xie, Shichun Chi (2019) Optimization Method for Irregular Piled Raft Foundation on Layered Soil Media. *Advances in Civil Engineering*, Vol. 2019, 1-15.

<https://doi.org/10.1155/2019/5713492>

22. Samorodov O. et al. (2022) New design of a combined pile raft foundation for a multi-storey building with determination of its main parameters. *Proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering*. Rahman and Jaksa (Eds). Australian Geomechanics Society, Sydney, Australia, 3493-3497

foundation. *Ukrainian Journal of Construction and Architecture*, No. 6 (012), pp. 90-98.

<https://doi.org/10.30838/J.BPSACEA.2312.271222.90.915>

11. Samorodov O.V. et al. (2021). Methodology for modeling the initial settlement of piles as part of a combined raft pile foundation in the PLAXIS 3D FOUNDATION software complex. *Scientific bulletin of construction*. Volume 105, No. 3, pp. 106-114

12. Comodromos, E. M., Papadopoulou, M. C. & Laloui, L. (2016). Contribution to the design methodologies of piled raft foundations under combined loadings. *Canadian Geotechnical Journal*, Vol. 53 (4), 559–577.

<http://doi.org/10.1139/cgj-2015-0251>

13. Chow, H. S. (2007). *Analysis of Piled-Raft Foundations with Piles of Different Lengths and Diameters*. Sydney: The University of Sydney.

<http://doi.org/10.1201/9781439833766.ch84>

14. Cunha, R. P., Poulos, H. G., & Small, J. C. (2001). Investigation of Design Alternatives for a Piled Raft Case History. *Journal of Geotechnical and Environmental Engineering*, 635-641.

[http://doi.org/10.1061/\(ASCE\)1090-0241\(2001\)127:8\(635\)](http://doi.org/10.1061/(ASCE)1090-0241(2001)127:8(635))

15. Hain, S., & Lee, I. (1978). The Analysis of Flexible Raft-Pile Systems. *Geotechnique*, 28 (1), 65-83.

<https://doi.org/10.1680/geot.1978.28.1.65>

16. Poulos, H. (1994). An Approximate Numerical Analysis of Pile-Raft Interaction. *Int. J. NAM Geomechs*, 18, 73-92.

<https://doi.org/10.1002/nag.1610180202>

17. Poulos, H. (2001). *Methods of Analysis of Piled Raft Foundations*. International Society of Soil Mechanics and Geotechnical Engineering

18. Reul, O., & Randolph, M. (2003). Piled Rafts in Over-consolidated Clay: Comparison of In situ Measurements and Numerical Analyses. *Geotechnique*, Vol. 53, No. 3, 301-315.

<https://doi.org/10.1680/geot.2003.53.3.301>

19. Shen, W., Chow, Y., & Yong, K. (1999). A Variational Solution for Vertically Loaded Pile Groups in an Elastic Half-space. *Geotechnique*, Vol. 49, No. 2, 199-213.

<https://doi.org/10.1680/geot.1999.49.2.199>

20. Simeneh Abate (2009). *Analysis and Parametric Study of Piled Raft Foundation Using Finite Element Based Software*. Addis Ababa University

21. Yunfei Xie, Shichun Chi (2019) Optimization Method for Irregular Piled Raft Foundation on Layered Soil Media. *Advances in Civil Engineering*, Vol. 2019, 1-15.

<https://doi.org/10.1155/2019/5713492>

22. Samorodov O. et al. (2022) New design of a combined pile raft foundation for a multi-storey building with determination of its main parameters. *Proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering*. Rahman and Jaksa (Eds). Australian Geomechanics Society, Sydney, Australia, 3493-3497